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★ Tutorial - 1

Q-1 Find REF and RREF of given matrix

(1) $\begin{bmatrix} 1 & -6 & 4 & 3 \\ 10 & 7 & 3 & 2 \end{bmatrix}$

→ $R_2 = R_2 - 10R_1$

$\begin{bmatrix} 1 & -6 & 4 & 3 \\ 0 & 67 & -37 & -28 \end{bmatrix}$

$R_2 (1/67)$

$\begin{bmatrix} 1 & -6 & 4 & 3 \\ 0 & 1 & -37/67 & -28/67 \end{bmatrix}$ ⇒ REF

$R_1 = R_1 + 6R_2$

$\begin{bmatrix} 1 & 0 & 46/67 & 33/67 \\ 0 & 1 & -37/67 & -28/67 \end{bmatrix}$ ⇒ RREF

(2) $\begin{bmatrix} 0 & -1 & 2 & 3 \\ 2 & 3 & 4 & 5 \\ 1 & 3 & -1 & 2 \\ 3 & 2 & 4 & 1 \end{bmatrix}$

$R_1 \leftrightarrow R_3$

$\begin{bmatrix} 1 & 3 & -1 & 2 \\ 2 & 3 & 4 & 5 \\ 0 & -1 & 2 & 3 \\ 3 & 2 & 4 & 1 \end{bmatrix}$

$R_2 = R_2 - 2R_1 \quad R_3 = R_3 - 3R_1$

$\begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & -3 & 6 & 1 \\ 0 & -1 & 2 & 3 \\ 0 & -7 & 7 & -5 \end{bmatrix}$

$R_3 \leftrightarrow R_2$

$\begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -3 & 6 & 1 \\ 0 & -7 & 7 & -5 \end{bmatrix}$

$R_2(-1)$

$$\begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & -3 & 6 & 1 \\ 0 & -7 & 7 & -5 \end{bmatrix}$$

$R_1 = R_1 - 3R_2$, $R_3 = R_3 + 3R_2$, $R_4 = R_4 + 7R_2$

$$\begin{bmatrix} 1 & 0 & -7 & -7 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 12 & 10 \\ 0 & 0 & 21 & 16 \end{bmatrix}$$

$R_3(1/12)$ $R_1(-1/7)$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 10/12 \\ 0 & 0 & 21 & 1/6 \end{bmatrix}$$

$R_2 = R_2 - 2R_2$, $R_1 = R_1 - R_3$, $R_4 = R_4 - 21R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 1/6 \\ 0 & 1 & 0 & 16/12 \\ 0 & 0 & 1 & 10/12 \\ 0 & 0 & 0 & -18/12 \end{bmatrix}$$

$R_4(-12/18)$

$$\begin{bmatrix} 1 & 0 & 0 & 1/6 \\ 0 & 1 & 0 & 16/12 \\ 0 & 0 & 1 & 10/12 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_1 = R_1 - \frac{1}{6}R_4$, $R_2 = R_2 - \frac{16}{12}R_4$, $R_3 = R_3 - \frac{10}{12}R_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \text{RREF}$$

(3) $$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

$\longrightarrow$ $R_2 = R_2 - 2R_1$, $R_3 = R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -5 \\ 0 & -5 & -7 \end{bmatrix}$$

$R_2(-1)$ $R_3(-1)$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 5 \\ 0 & 5 & 7 \end{bmatrix}$$

$R_3 = R_3 - 5R_2$, $R_1 = R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 & -7 \\ 0 & 1 & 5 \\ 0 & 0 & -18 \end{bmatrix}$$

$R(-\frac{1}{18})$

$$\begin{bmatrix} 1 & 0 & -7 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 = R_2 - 5R_3$, $R_1 = R_1 + 7R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \text{RREF}$$

(4)
$$\begin{bmatrix} 3 & 4 & -2 & 1 \\ 5 & 8 & 4 & 2 \\ 8 & 12 & 2 & 3 \\ 12 & 20 & 6 & 5 \end{bmatrix}$$

$\rightarrow$ $R_1(1/3)$

$$\begin{bmatrix} 1 & 4/3 & -2/3 & 1/3 \\ 5 & 8 & 4 & 2 \\ 8 & 12 & 2 & 3 \\ 12 & 20 & 6 & 5 \end{bmatrix}$$

$R_2 = R_2 - 5R_1$, $R_3 = R_3 - 8R_1$, $R_4 = R_4 - 12R_1$

$$\begin{bmatrix} 1 & 4/3 & -2/3 & 1/3 \\ 0 & +4/3 & 22/3 & 1/3 \\ 0 & 4/3 & 22/3 & 1/3 \\ 0 & 4 & 14 & 1 \end{bmatrix}$$

$R_3 = R_3 - R_2$

$$\begin{bmatrix} 1 & 4/3 & -2/3 & 1/3 \\ 0 & 4/3 & 22/3 & 1/3 \\ 0 & 0 & 0 & 0 \\ 0 & 4 & 14 & 1 \end{bmatrix}$$

$R_3 \leftrightarrow R_4$

$$\begin{bmatrix} 1 & 4/3 & -2/3 & 1/3 \\ 0 & 4/3 & 22/3 & 1/3 \\ 0 & 4 & 14 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 = R_1 - R_2$

$$\begin{bmatrix} 1 & 0 & -8 & 0 \\ 0 & 4/3 & 22/3 & 1/3 \\ 0 & 4 & 14 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2(3/4)$

$$\begin{bmatrix} 1 & 0 & -8 & 0 \\ 0 & 1 & 22/4 & 1/4 \\ 0 & 4 & 14 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 = R_3 - 4R_2$

$$\begin{bmatrix} 1 & 0 & -8 & 0 \\ 0 & 1 & 22/4 & 1/4 \\ 0 & 0 & -8 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3(-1/8)$

$$\begin{bmatrix} 1 & 0 & -8 & 0 \\ 0 & 1 & 22/4 & 1/4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 = R_1 + 8R_3, \quad R_2 = R_2 - 22/4\, R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1/4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \text{RREF}$$

Q-2 Find the rank of matrix using defination :

(i) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$

→ to find rank of matrix using defination,

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{vmatrix}$$

$$= 1(21-20) - 2(14-12) + 3(10-9)$$

$$= 1 - 4 + 3$$

$$= 0$$

$$\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 3 - 4 = -1 \neq 0$$

Therefore rank of matrix = 2

$S(A) = 2$

(2)

$$\begin{vmatrix} 1 & 2 & -1 & -4 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & -7 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 2 & -1 \\ 2 & 4 & 3 \\ -1 & -2 & 6 \end{vmatrix} = 1(24+6) - 2(12+3) + -1(-4+4)$$

$$= 30 - 30 - 0 = 0$$

$$= -15 \neq 0$$

$$\begin{vmatrix} 2 & -1 & -4 \\ 4 & 3 & 5 \\ -2 & 6 & -7 \end{vmatrix} = 2(-21-30) + 1(-28+10) - 4(24+6)$$

$$= -102 - 18 - 120 \neq 0$$

$$\begin{vmatrix} -1 & -4 & 1 \\ 3 & 5 & 2 \\ 6 & -7 & -1 \end{vmatrix} = -1(-5+14) + 4(-3-12) + 1(-21-30)$$

$$= -9 - 60 - 51$$

$$\neq 0$$

Rank of matrix = 3

(3)

$$\begin{vmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & 2 \\ 3 & 1 & 0 \end{vmatrix} = 1(0-2) - 1(0-6) - 1(1+3)$$

$$= -2 + 6 - 4$$

$$= 0$$

$$\begin{vmatrix} 1 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & 0 & 1 \end{vmatrix} = 1(2+1) + 1(-1+1) + 1(0-2)$$

$$= 2 - 2 = 0$$

$$\begin{vmatrix} -1 & 1 & 1 \\ 2 & -1 & 1 \\ 0 & 1 & 3 \end{vmatrix} = -1(-3-1) - 1(6-0) + 1(2+0)$$
$$= 4 - 6 + 2$$
$$= 0$$

$$\begin{vmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ 1 & 3 & 1 \end{vmatrix} = 1(1+3) - 1(-1+1) + 1(-3-1)$$
$$= 4 - 4$$
$$= 0$$

$S(A) \neq 3$

$$\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -1 - 1 = -2 \neq 0$$

therefore rank of matrix = 2

$S(A) = 2$

(4) $\begin{bmatrix} 1 & -2 & 3 \\ -1 & 1 & -2 \\ 2 & -1 & 3 \end{bmatrix}$

$$\begin{vmatrix} 1 & -2 & 3 \\ -1 & 1 & -2 \\ 2 & -1 & 3 \end{vmatrix} = 1(3-2) + 2(\frac{-3+4}{-2}) + 3(1-2)$$
$$= 1 - 2 - 3 \quad 1 + 2 - 3$$
$$= 0$$

$$\begin{vmatrix} 1 & -2 \\ -1 & 1 \end{vmatrix} = 1 - 2 = -1 \neq 0$$

therefore rank of matrix = 2

$S(A) = 2$

Q-3 Converting given matrix into RREF and find rank of matrix:

(1) $\begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix}$

$R_2 \leftrightarrow R_3$ $\quad$ $R_1 \leftrightarrow R_3$

$\begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 5 & 3 & 14 & 4 \end{bmatrix}$

$R_3 = R_3 - 5R_1$

$\begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 8 & 4 & 4 \end{bmatrix}$

$R_3 = R_3 - 8R_2$, $R_1 = R_1 + R_2$

~~$R_1 = R_1 - R_2$~~, ~~$R_3(1/4)$~~

$\begin{bmatrix} 1 & 0 & 4 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & -12 & -4 \end{bmatrix}$

$R_3(-1/12)$

$\begin{bmatrix} 1 & 0 & 4 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1/3 \end{bmatrix}$

$R_2 = R_2 - 2R_3$, $R_1 = R_1 - 4R_3$

$\begin{bmatrix} 1 & 0 & 0 & 1/3 \\ 0 & 1 & 0 & 1/3 \\ 0 & 0 & 1 & 1/3 \end{bmatrix}$

There are 3 non zero rows
There fore rank of matrix = 3
$S(A) = 3$

(2)
$$\begin{bmatrix} 0 & 1 & 3 & -2 \\ 0 & 4 & -21 & 3 \\ 0 & 0 & 2 & 1 \\ 0 & 5 & -3 & 4 \end{bmatrix}$$

$R_2 = R_2 - 4R_1$, $R_4 = R_4 - 5R_1$

$$\begin{bmatrix} 0 & 1 & 3 & -2 \\ 0 & 0 & -13 & 11 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & -18 & 14 \end{bmatrix}$$

$R_2 \cdot (-1/13)$

$$\begin{bmatrix} 0 & 1 & 3 & -2 \\ 0 & 0 & 1 & -11/13 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & -18 & 14 \end{bmatrix}$$

$198/13 + 14$

$1/2$

182

198

380

$2 + 33/13$

$7/13$

$R_3 = R_3 - 2R_2$, $R_1 = R_1 - 3R_2$, $R_4 = R_4 + 18R_2$

$$\begin{bmatrix} 0 & 1 & 0 & 7/13 \\ 0 & 0 & 1 & -11/13 \\ 0 & 0 & 0 & -35/13 \\ 0 & 0 & 0 & 380/13 \end{bmatrix}$$

$R_1 = R_1 - 7/13 R_3$, $R_2 = R_2 + 11/13 R_3$

$R_4 = R_4 - 380/13 R_3$

$28/13$

$R_3 (13/35)$

$$\begin{bmatrix} 0 & 1 & 0 & 7/13 \\ 0 & 0 & 1 & -11/13 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 380/13 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \text{RREF}$$

There are 3 non zero rows
Therefore rank of matrix = 3
$S(A) = 3$

(3)
$$\begin{bmatrix} 1 & 2 & 3 & -1 \\ -2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$\longrightarrow$ $R_2 = R_2 + 2R_1$, $R_3 = R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 3 & 3 & -3 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_2(1/3)$ $R_3(1/2)$

$$\begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & -1 & -1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_3 = R_3 + R_2$, $R_4 = R_4 - R_2$

$R_1 = R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

There are two non zero rows
Therefore rank of matrix = 2

$S(A) = 2$

(4) $\begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$

$R_2 = R_2 - 3R_1$, $R_3 = R_3 + 2R_1$

$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & -2 & 1 & 5 \\ 0 & 7 & 2 & 3 \end{bmatrix}$

$R_2(-1/2)$

$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & -1/2 & -5/2 \\ 0 & 7 & 2 & 3 \end{bmatrix}$

$3 + 35/2$

$R_1 = R_1 - 2R_2$, $R_3 = R_3 - 7R_2$

$2 - 7/2$

$-3/2$

$\begin{bmatrix} 1 & 0 & 1 & 4 \\ 0 & 1 & -1/2 & -5/2 \\ 0 & 0 & 11/2 & 41/2 \end{bmatrix}$

$R_3(2/11)$

$\begin{bmatrix} 1 & 0 & 1 & 4 \\ 0 & 1 & -1/2 & -5/2 \\ 0 & 0 & 1 & 41/11 \end{bmatrix}$

$R_1 = R_1 - R_3$, $R_2 = R_2 + 1/2 R_3$

$\begin{bmatrix} 1 & 0 & 0 & 3/11 \\ 0 & 1 & 0 & -3/11 \\ 0 & 0 & 1 & 41/11 \end{bmatrix}$

$\frac{41}{11 \times 2} + \frac{-5}{2}$

$\frac{-6}{22}$

There are 3 non zero rows

Therefore rank of matrix = 3

$S(A) = 3$

Q-4 Convert the given matrix into Normal form and find the rank of matrix.

(1) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$

$R_2 = R_2 - 2R_1$, $R_3 = R_3 - 3R_1$

$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -5 & 2 \\ 0 & -5 & 0 \end{bmatrix}$

$R_2(-1/5)$

$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -2/5 \\ 0 & -5 & 0 \end{bmatrix}$

$R_1 = R_1 - R_2$, $R_3 = R_3 + 5R_2$

$\begin{bmatrix} 1 & 0 & 7/5 \\ 0 & 1 & -2/5 \\ 0 & 0 & -2 \end{bmatrix}$

$R_3(-1/2)$

$\begin{bmatrix} 1 & 0 & 7/5 \\ 0 & 1 & -2/5 \\ 0 & 0 & 1 \end{bmatrix}$

$R_2 = R_2 + 2/5 R_3$, $R_1 = R_1 - 7/5 R_3$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

I_3

∴ Rank of matrix = 3

$S(A) = 3$

(2)
$$\begin{bmatrix} 1 & -1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

$\longrightarrow$ $R_2 = R_2 - 4R_1$

$$\begin{bmatrix} 1 & -1 & 2 & -3 \\ 0 & 5 & -8 & 14 \\ 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

$C_2 = C_2 + C_1$, $C_3 = C_3 - 2C_1$
$C_4 = C_4 + 3C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & -8 & 14 \\ 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

$R_3 \leftrightarrow R_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 3 & 0 & 4 \\ 0 & 5 & -8 & 14 \end{bmatrix}$$

$R_3 = R_3 - 3R_2$, $R_4 = R_4 - 5R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & -8 & 4 \end{bmatrix}$$

$C_4 = C_4 - 2C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & -8 & 4 \end{bmatrix}$$

$R_4 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -8 & 4 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

$C_3(-1/8)$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

$C_4 = C_4 - 4C_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

$R_4(-1/2)$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I_3$$

$\therefore$ Rank of matrix $= 3$

$S(A) = 3$

(3) $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & 4 & 0 & -1 \\ -1 & 0 & -2 & 7 \end{bmatrix}$

$R_2 = R_2 - 3R_1$, $R_4 = R_4 + R_1$

$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -2 & 3 & -10 \\ 0 & 2 & -3 & 10 \end{bmatrix}$

$R_3 = R_3 + R_2$

$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -2 & 3 & -10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$C_2 = C_2 - 2C_1$, $C_3 = C_3 + C_1$

$C_4 = C_4 - 3C_1$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -2 & 3 & -10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$C_2(-1/2)$, $C_3(1/3)$

$C_4(-1/10)$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$C_4 = C_4 - C_2$, $C_3 = C_3 - C_2$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

I_2

$\therefore$ Rank of matrix = 2

$S(A) = 2$

(4) $\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 1 & -1 & 4 & 0 \\ -2 & 2 & 8 & 0 \end{bmatrix}$

$\longrightarrow$ $R_3 = R_3 - R_1$, $R_4 = R_4 + 2R_1$

$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & -1 & 2 & -1 \\ 0 & 2 & 12 & 2 \end{bmatrix}$

$C_3 = C_3 - 2C_1$, $C_4 = C_4 - C_1$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & -1 & 2 & -1 \\ 0 & 2 & 12 & 2 \end{bmatrix}$

$R_3 = R_3 + R_2$, $R_4 = R_4 - 2R_2$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 \end{bmatrix}$

$R_3 \leftrightarrow R_4$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$C_3 = C_3 + 2C_1$, $C_4 = C_4 - C_3$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$C_3(1/16)$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

I_3

$\therefore$ Rank of matrix = 3

$S(A) = 3$

Q-5 Find the inverse of the following Matrics using a Gauss-Jordan method.

(1) $\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

$\longrightarrow \left[\begin{array}{ccc:ccc} 3 & 2 & 6 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 2 & 2 & 5 & 0 & 0 & 1 \end{array}\right]$

$R_2 \leftrightarrow R_1$

$\left[\begin{array}{ccc:ccc} 1 & 1 & 2 & 0 & 1 & 0 \\ 3 & 2 & 6 & 1 & 0 & 0 \\ 2 & 2 & 5 & 0 & 0 & 1 \end{array}\right]$

$R_2 = R_2 - 3R_1, \quad R_3 = R_3 - 2R_1$

$\left[\begin{array}{ccc:ccc} 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & -3 & 0 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{array}\right]$

$R_2(-1)$

$\left[\begin{array}{ccc:ccc} 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 3 & 0 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{array}\right]$

$R_1 = R_1 - R_2$

$\left[\begin{array}{ccc:ccc} 1 & 0 & 2 & 1 & -2 & 0 \\ 0 & 1 & 0 & -1 & 3 & 0 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{array}\right]$

$R_1 = R_1 - 2R_3$

$\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & 2 & -2 \\ 0 & 1 & 0 & -1 & 3 & 0 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{array}\right]$

$A^{-1} = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

(2) $\begin{bmatrix} 4 & 1 & -1 \\ 2 & 0 & -1 \\ 1 & -1 & 3 \end{bmatrix}$

$\rightarrow \left[\begin{array}{ccc:ccc} 4 & 1 & -1 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ 1 & -1 & 3 & 0 & 0 & 1 \end{array}\right]$

$R_3 \leftrightarrow R_1$

$\left[\begin{array}{ccc:ccc} 1 & -1 & 3 & 0 & 0 & 1 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ 4 & 1 & -1 & 1 & 0 & 0 \end{array}\right]$

$R_2 = R_2 - 2R_1, \; R_3 = R_3 - 4R_1$

$\left[\begin{array}{ccc:ccc} 1 & -1 & 3 & 0 & 0 & 1 \\ 0 & 2 & -7 & 0 & 1 & -2 \\ 0 & 5 & -13 & 1 & 0 & -4 \end{array}\right]$

$R_2 (1/2)$

$\left[\begin{array}{ccc:ccc} 1 & -1 & 3 & 0 & 0 & 1 \\ 0 & 1 & -7/2 & 0 & 1/2 & -1 \\ 0 & 5 & -13 & 1 & 0 & -4 \end{array}\right]$

$R_1 = R_1 + R_2, \; R_3 = R_3 - 5R_2$

$\left[\begin{array}{ccc:ccc} 1 & 0 & -1/2 & 0 & 1/2 & 0 \\ 0 & 1 & -7/2 & 0 & 1/2 & -1 \\ 0 & 0 & 9/2 & 1 & -5/2 & 1 \end{array}\right]$

$R_3 (2/9)$

$\left[\begin{array}{ccc:ccc} 1 & 0 & -1/2 & 0 & 1/2 & 0 \\ 0 & 1 & -7/2 & 0 & 1/2 & -1 \\ 0 & 0 & 1 & 2/9 & -5/9 & 2/9 \end{array}\right]$

$R_1 = R_1 + 1/2 R_3, \; R_2 = R_2 + 7/2 R_3$

$\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1/9 & 2/9 & -1/9 \\ 0 & 1 & 0 & 7/9 & 22/9 & -16/9 \\ 0 & 0 & 1 & 2/9 & -5/9 & 2/9 \end{array}\right]$

$A^{-1} = \begin{bmatrix} 1/9 & 2/9 & -1/9 \\ 7/9 & 22/9 & -16/9 \\ 2/9 & -5/9 & 2/9 \end{bmatrix}$

(3) $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

$\rightarrow \left[\begin{array}{ccc:ccc} 3 & -3 & 4 & 1 & 0 & 0 \\ 2 & -3 & 4 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array}\right]$

$R_1 = R_1 - R_2$

$\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 2 & -3 & 4 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array}\right]$

$R_2 = R_2 - 2R_1$

$\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & -3 & 4 & -2 & 3 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array}\right]$

$R_3(-1)$

$\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & -3 & 4 & -2 & 3 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 \end{array}\right]$

$R_2 \leftrightarrow R_3$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 \\ 0 & -3 & 4 & -2 & 3 & 0 \end{array}\right]$$

$R_3 = R_3 + 3R_2$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -2 & 3 & -3 \end{array}\right]$$

$R_2 = R_2 + R_3$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -2 & 3 & -4 \\ 0 & 0 & 1 & -2 & 3 & -3 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

(4) $\begin{bmatrix} 5 & -1 & 5 \\ 0 & 2 & 0 \\ -5 & 3 & -15 \end{bmatrix}$

$\longrightarrow \left[\begin{array}{ccc:ccc} 5 & -1 & 5 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ -5 & 3 & -15 & 0 & 0 & 1 \end{array}\right]$

$R_3 = R_3 + R_1$

$\left[\begin{array}{ccc:ccc} 5 & -1 & 5 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 2 & -10 & 1 & 0 & 1 \end{array}\right]$

$R_1(1/5)$

$\left[\begin{array}{ccc:ccc} 1 & -1/5 & 1 & 1/5 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 2 & -10 & 1 & 0 & 1 \end{array}\right]$

$R_2(1/2)$

$\left[\begin{array}{ccc:ccc} 1 & -1/5 & 1 & 1/5 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1/2 & 0 \\ 0 & 2 & -10 & 1 & 0 & 1 \end{array}\right]$

$R_3 = R_3 - 2R_1$, $R_1 = R_1 + 1/5\, R_2$

$\left[\begin{array}{ccc:ccc} 1 & 0 & 1 & 1/5 & 1/10 & 0 \\ 0 & 1 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & -10 & 1 & -1 & 1 \end{array}\right]$

$R_3(-1/10)$

$\left[\begin{array}{ccc:ccc} 1 & 0 & 1 & 1/5 & 1/10 & 0 \\ 0 & 1 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & -1/10 & 1/10 & -1/10 \end{array}\right]$

$R_1 = R_1 - R_3$

$\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 3/10 & 0 & 1/10 \\ 0 & 1 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & -1/10 & 1/10 & -1/10 \end{array}\right]$

$A^{-1} = \begin{bmatrix} 3/10 & 0 & 1/10 \\ 0 & 1/2 & 0 \\ -1/10 & 1/10 & -1/10 \end{bmatrix}$